# Formulaire de trigonométrie

*Les formules sont valables sous réserve qu'elles soient définies.*

## Fonctions circulaires

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$e^{ix} = \cos x + i \sin x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\cos a \cos b = \tfrac{1}{2}\big(\cos(a+b) + \cos(a-b)\big)$$

$$\sin a \sin b = \tfrac{1}{2}\big(\cos(a-b) - \cos(a+b)\big)$$

$$\sin a \cos b = \tfrac{1}{2}\big(\sin(a+b) + \sin(a-b)\big)$$

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2}$$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$$

$$\tan p + \tan q = \frac{\sin(p+q)}{\cos p \cos q}$$

$$\tan p - \tan q = \frac{\sin(p-q)}{\cos p \cos q}$$

$$\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a$$

$$\sin(2a) = 2 \sin a \cos a$$

$$\cos^2 a = \frac{1 + \cos(2a)}{2} \qquad \sin^2 a = \frac{1 - \cos(2a)}{2}$$

$$\cos(2a) = \frac{1 - \tan^2 a}{1 + \tan^2 a} \qquad \sin(2a) = \frac{2\tan a}{1 + \tan^2 a}$$

$$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$$

$$\forall x \in [-1, 1],\ \operatorname{Arcsin} x + \operatorname{Arccos} x = \frac{\pi}{2}$$

$$\forall a \in \mathbb{R}^*,\ \operatorname{Arctan} a + \operatorname{Arctan} \frac{1}{a} = \operatorname{sgn} a \frac{\pi}{2}$$

$$\forall x \in \,]-1, 1[,\ \operatorname{Arcsin}' x = \frac{1}{\sqrt{1 - x^2}}$$

$$\forall x \in \,]-1, 1[,\ \operatorname{Arccos}' x = \frac{-1}{\sqrt{1 - x^2}}$$

$$\forall x \in \mathbb{R},\ \operatorname{Arctan}' x = \frac{1}{1 + x^2}$$

## Fonctions hyperboliques

$$\boxed{\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1}$$

$$\operatorname{ch} x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sh} x = \frac{e^{x} - e^{-x}}{2}$$

$$e^{x} = \operatorname{ch} x + \operatorname{sh} x$$

$$\operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{ch}(a-b) = \operatorname{ch} a \operatorname{ch} b - \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{ch} a \operatorname{sh} b$$

$$\operatorname{sh}(a-b) = \operatorname{sh} a \operatorname{ch} b - \operatorname{ch} a \operatorname{sh} b$$

$$\operatorname{ch} a \operatorname{ch} b = \tfrac{1}{2}\big(\operatorname{ch}(a+b) + \operatorname{ch}(a-b)\big)$$

$$\operatorname{sh} a \operatorname{sh} b = \tfrac{1}{2}\big(\operatorname{ch}(a+b) - \operatorname{ch}(a-b)\big)$$

$$\operatorname{sh} a \operatorname{ch} b = \tfrac{1}{2}\big(\operatorname{sh}(a+b) + \operatorname{sh}(a-b)\big)$$

$$\operatorname{sh} p + \operatorname{sh} q = 2 \operatorname{sh} \frac{p+q}{2} \operatorname{ch} \frac{p-q}{2}$$

$$\operatorname{sh} p - \operatorname{sh} q = 2 \operatorname{ch} \frac{p+q}{2} \operatorname{sh} \frac{p-q}{2}$$

$$\operatorname{ch} p + \operatorname{ch} q = 2 \operatorname{ch} \frac{p+q}{2} \operatorname{ch} \frac{p-q}{2}$$

$$\operatorname{ch} p - \operatorname{ch} q = 2 \operatorname{sh} \frac{p+q}{2} \operatorname{sh} \frac{p-q}{2}$$

$$\operatorname{ch}(2a) = \operatorname{ch}^2 a + \operatorname{sh}^2 a = 2\operatorname{ch}^2 a - 1 = 1 + 2\operatorname{sh}^2 a$$

$$\operatorname{sh}(2a) = 2 \operatorname{sh} a \operatorname{ch} a$$

$$\operatorname{ch}^2 a = \frac{1 + \operatorname{ch}(2a)}{2} \qquad \operatorname{sh}^2 a = \frac{\operatorname{ch}(2a) - 1}{2}$$